

Duality in SUSY $SU(N)$ with an Antisymmetric Tensor

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We present a dual description for $SU(N)$ supersymmetric gauge theory with an antisymmetric tensor and fundamentals, and no superpotential. This duality is derived from the dualities of Seiberg. Under a perturbation of the superpotential, the dual theory breaks supersymmetry at tree level.

One year ago, a remarkable feature of gauge field theories was discovered [1]. Certain $\mathcal{N}=1$ supersymmetric field theories were shown by Seiberg to have a completely equivalent description at large distances in terms of a different SUSY gauge theory, with a different gauge group and matter content. For reviews and lists of references, see [2]. For earlier work on these SUSY theories see [3-5]. This duality is a generalization of the one of Montonen and Olive of $\mathcal{N}=4$ [6] and some $\mathcal{N}=2$ SUSY gauge theories [7].

The original examples in [1] included $SU(N)$, $SO(N)$ and $Sp(N)$ gauge theories with fundamentals. They were further studied in [8,9]. It is not straightforward to find dual descriptions for other gauge groups and matter content. A dual description for SUSY G_2 gauge theory with fundamentals was found in [10]. It was extended to other theories in [11]. A wealth of partial results on dualizing many other theories were found in [12-18], in an approach which requires to perturb the theory by a superpotential in order to perform the duality.

In this letter, the theory that we consider is an $SU(N)$ supersymmetric gauge theory without a superpotential. The matter fields are chiral superfields transforming in one antisymmetric tensor representation A , F fundamentals Q and $N+F-4$ antifundamentals \overline{Q} . That is, there is F extra flavors of Q and \overline{Q} beyond the necessary antifundamentals to cancel the $SU(N)^3$ anomaly. We present a dual description for this model for N odd, which we refer to as the electric theory. A dual for this model with a superpotential was obtained in [15] for N even. We derive this duality from the elementary dualities of Seiberg following the idea of [15] of *deconfining* the antisymmetric tensor, by introducing an auxiliary gauge group. We also propose a dual description for $SU(N)$ with an antisymmetric tensor and a conjugate antisymmetric tensor, fundamentals and antifundamentals, which we do not analyze in detail. We then study some of the features of the model with one antisymmetric tensor in detail. One feature is that a baryon operator is an elementary field in the dual description. Also, this duality flows to the SU and Sp dualities of [1] and that of [15]. Another interesting feature is that this model was shown in [4] to break supersymmetry dynamically. This aspect was further studied in [19-22]. The novelty here is that SUSY breaking is studied at weak coupling in the dual description.

We first describe the electric theory. Under the continuous non-anomalous $SU(F) \times SU(N+F-4) \times U(1)_1 \times U(1)_2 \times U(1)_R$ symmetries, the fields transform as:

	$SU(N)$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$	
A	\square	1	1	0	$-2F$	$\frac{-12}{N}$	(1)
Q	\square	\square	1	1	$N-F$	$2 - \frac{6}{N}$	
\overline{Q}	$\overline{\square}$	1	$\overline{\square}$	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$	

There is a two parameter family of R-symmetries; our choice of $U(1)_R$ is for convenience.

The flat directions can be conveniently described by the following gauge invariant chiral operators: mesons $M \equiv Q\bar{Q}$ and $H \equiv A\bar{Q}\bar{Q}$; baryons $\bar{B} \equiv \bar{Q}^N$ ($F \geq 4$) and $B_k \equiv Q^k A^{\frac{N-k}{2}}$ (N and k both odd or both even, $k \leq \min(N, F)$). These operators are not all independent [20], but classically constrained. We will not discuss these constraints. When M gets an expectation value of rank r , the theory is higgsed to $SU(N-r)$ with, as the remaining matter content, an antisymmetric tensor \hat{A} , $N-r+F-4$ antifundamentals $\hat{\bar{Q}}$ and still F fundamentals \hat{Q} (among which r are coming from A). Similarly, when H gets an expectation value of rank $2r$, the group is higgsed to $SU(N-2r)$ with an antisymmetric tensor \hat{A} , $N-2r+F-4$ antifundamentals $\hat{\bar{Q}}$ and F fundamentals \hat{Q} remaining. When B_k gets an expectation value, the theory is higgsed to $Sp(\frac{N-k}{2})$ with $N-k+2F-4$ fundamentals \hat{Q} remaining, coming from \bar{Q} and Q . When \bar{B} gets an expectation value, the theory is completely higgsed.

For completeness, we briefly summarize the results of [20] in our notation. For $F \geq 3$, by holomorphy, the symmetries and weak coupling, no superpotential can be generated dynamically. For $F = 0$ and N odd, there is no invariant that can appear in the superpotential, which remains $W = 0$; upon adding a tree level term λH to the superpotential, it was shown that the theory has no supersymmetric vacuum [4,19,20]. For $F = 0$ and N even, a superpotential $(\Lambda^{2N+3}/(B_0 \text{Pf } H))^{1/3}$ is generated by gluino condensation in an $Sp(2)$ subgroup of $SU(N)$. For $F = 1$, a superpotential is generated by gluino condensation in an $Sp(1)$ subgroup of $SU(N)$; it is $(\Lambda^{2N+2}/(B_1 \text{Pf } H))^{1/2}$ (for N odd) and $(\Lambda^{2N+2}/(B_0 M H^{(N-4)/2}))^{1/2}$ (for N even). For $F = 2$, a superpotential is generated by instantons; it is $\Lambda^{2N+1}/(B_1 M H^{(N-3)/2})$ (for N odd) and $\Lambda^{2N+1}/(B_0 M^2 H^{-1} \text{Pf } H + B_2 \text{Pf } H)$ (for N even). For $F = 3$, the singular classical moduli space is smoothed out quantum mechanically, as the classical constraint is modified to $B_1 M^2 H^{-1} \text{Pf } H - B_3 \text{Pf } H = \Lambda^{2N}$ (for N odd) and $B_0 M^3 H^{(N-4)/2} + B_2 M H^{(N-2)/2} = \Lambda^{2N}$ (for N even). For $F \geq 4$, the classical and the quantum moduli spaces of vacua are the same. For $F = 4$, the theory is one of massless mesons and baryons at the origin of the moduli space. It consists of the independent gauge invariant degrees of freedom M , H , B_1 , B_3 and \bar{B} (for N odd) and of M , H , B_0 , B_2 , B_4 and \bar{B} (for N even). This satisfies the 't Hooft anomaly matching conditions. Their interaction can be described by the confining superpotentials $(B_1 M^3 H^{(N-3)/2} + B_3 M H^{(N-1)/2} + \bar{B} B_1 B_3)/\Lambda^{2N-1}$ (for N odd) and $(B_0 M^4 H^{-2} \text{Pf } H + B_2 M^2 H^{-1} \text{Pf } H + B_4 \text{Pf } H + \bar{B} B_0 B_4 + \bar{B} B_2^2)/\Lambda^{2N-1}$ (for N even), and the resulting equations of motion yield constraints on the expectation values of the fields.

When $F \geq 5$, the theory at the origin of the moduli space is in a non-Abelian Coulomb phase. The main result of this paper is that for N odd, the $SU(N)$ theory described above, which we will call electric, has a dual description in terms of an $SU(F-3) \times Sp(F-4)$ gauge theory with five species of dual quark superfields: a field x transforming as a fundamental under both gauge groups, a conjugate antisymmetric tensor \bar{a} , a fundamental p and F antifundamentals \bar{q} of $SU(F-3)$ and also $N+F-4$ fundamentals l of $Sp(F-4)$. Furthermore, this dual magnetic theory, later referred to as the first dual, contains elementary gauge singlet fields M , H and B_1 . The global non-anomalous symmetry is the same as in the electric theory and the transformation properties of these fields are listed below.

	$SU(F-3)$	$Sp(F-4)$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$	
x	\square	\square	1	1	$\frac{-F}{F-3}$	0	-1	
p	\square	1	1	1	$\frac{-F}{F-3}$	NF	6	
\bar{a}	$\overline{\square}$	1	1	1	$\frac{2F}{F-3}$	0	4	
\bar{q}	$\overline{\square}$	1	$\overline{\square}$	1	$\frac{3}{F-3}$	$-N$	0	(2)
l	1	\square	1	$\overline{\square}$	$\frac{F}{N+F-4}$	0	1	
M	1	1	\square	\square	$\frac{N-4}{N+F-4}$	N	2	
H	1	1	1	$\overline{\square}$	$\frac{-2F}{N+F-4}$	0	0	
B_1	1	1	\square	1	1	$N(1-F)$	-4.	

This magnetic theory has the superpotential

$$W = M\bar{q}lx + Hll + B_1p\bar{q} + \bar{a}x^2. \quad (3)$$

It is the most general superpotential allowed by the symmetries, holomorphy, and smoothness near the origin in field space. Note that the form of W , along with the identification of the operators M , H and B_1 of the dual with those of the electric theory, the $U(1)$ charges are determined. Under this charge assignment, the three $U(1)$ s are anomaly free in both the electric and magnetic theory. The 't Hooft anomaly matching conditions for the full $SU(F) \times SU(N+F-4) \times U(1)_1 \times U(1)_2 \times U(1)_R$ global symmetry are satisfied. These conditions are clearly satisfied for any complex numbers F and N . But the magnetic theory presented here is certainly not valid for N even (see the mapping of operators below) and therefore more checks are required to establish the duality for N odd. One route is to check the maps of operators, perturbations of the superpotential, flat directions, etc.

We will return to this briefly later. Alternatively, this duality can be derived from the elementary dualities of Seiberg.

To this end, consider an $Sp(\frac{N-3}{2})$ SUSY gauge theory (for N odd) with $N+1$ fundamentals, y_i and z , and N singlets \bar{P}^i , $i = 1, \dots, N$. This theory confines [9] and yields a superpotential $W = y^N z = A^{\frac{N-1}{2}} P$ for the gauge invariant fields $A_{ij} \equiv y_i y_j$ and $P_i \equiv z y_i$. Add to the superpotential a coupling $z y_i \bar{P}^i$, which is a mass term for P_i and \bar{P}^i . This coupling breaks the $SU(N+1) \times SU(N)$ flavor symmetry to $SU(N) \times U(1)$. Integrating out P_i and \bar{P}^i , we get a theory with $\frac{N(N-1)}{2}$ singlets A and no superpotential; there is no constraint on the light fields A . Now consider gauging the $SU(N)$ flavor symmetry, under which A is an antisymmetric tensor; introduce more $Sp(\frac{N-3}{2})$ singlets Q and \bar{Q} , which are fundamentals and antifundamentals of $SU(N)$, to cancel the $SU(N)^3$ anomaly. Therefore, the $SU(N) \times Sp(\frac{N-3}{2})$ expanded theory just described is equivalent to the electric theory (1). The charge assignments in the expanded theory follow from the relations $W = y^N z + z y \bar{P}$, $A_{ij} \equiv y_i y_j$ and $P_i \equiv z y_i$, to obtain:

	$SU(N)$	$Sp(\frac{N-3}{2})$	$SU(F)$	$SU(N+F-4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$	
y	\square	\square	1	1	0	$-F$	$\frac{-6}{N}$	
z	1	\square	1	1	0	FN	$\frac{8}{N}$	(4)
\bar{P}	$\bar{\square}$	1	1	1	0	$F(1-N)$	$-6 + \frac{6}{N}$	
Q	\square	1	\square	1	1	$N-F$	$2 - \frac{6}{N}$	
\bar{Q}	$\bar{\square}$	1	1	\square	$\frac{-F}{N+F-4}$	F	$\frac{6}{N}$	

The $SU(N)$ gauge group has $N+F-3$ flavors, and is in a non-Abelian Coulomb phase precisely for $F \geq 5$; it can be dualized by the $SU(N)$ duality of [1]. The result is a dual description in terms of an $SU(F-3) \times Sp(\frac{N-3}{2})$ gauge theory with the following matter content:

	$SU(F-3)$	$Sp(\frac{N-3}{2})$	$SU(F)$	$SU(N+F-4)$	
\bar{x}	$\bar{\square}$	\square	1	1	
p	\square	1	1	1	
\bar{q}	$\bar{\square}$	1	$\bar{\square}$	1	
q	\square	1	1	$\bar{\square}$	
\bar{l}	1	\square	1	\square	
$M = Q\bar{Q}$	1	1	\square	\square	
$B_1 = QA^{\frac{N-1}{2}}$	1	1	\square	1.	

The superpotential is

$$W = Mq\bar{q} + B_1 p\bar{q} + \bar{l}xq. \quad (6)$$

Now observe that the $Sp(\frac{N-3}{2})$ gauge group has $N-7+2F$ fundamentals, so that precisely for $F \geq 5$, it is in a non-Abelian Coulomb phase and can be dualized by the Sp duality of [1]. The result after integrating out the massive fields is the dual description presented above (2).

Going back to the first dual, note that its $SU(F-3)$ gauge group has matter content in an antisymmetric tensor, fundamental and antifundamental representations, and thus can be dualized with the duality presented in this letter; this procedure can be iterated n times to yield dual descriptions with gauge group $SU(F-3) \times \prod_{i=0}^n Sp(F-4)_i$. One generalization of this construction is to dualize $SU(N)$ (for N odd) with an antisymmetric tensor \square , a conjugate antisymmetric tensor $\bar{\square}$ and F flavors of fundamentals \square and antifundamentals $\bar{\square}$, without a superpotential. This model with a superpotential was studied in [18]. We suggest that the iteration procedure just mentioned could account for the multiple vacua obtained in [18] of the form $SU(n_0) \times \prod_i Sp(n_i)$.

We now study the features of the $SU(N)$ model with only one antisymmetric tensor in more detail. The baryon operators of the electric theory are mapped to baryons of the magnetic theory in the following way:

$$\bar{Q}^N \rightarrow p(xl)^{F-4} \quad Q^k A^{\frac{N-k}{2}} \rightarrow \tilde{B}_{F-k} \equiv \bar{q}^{F-k} \bar{a}^{\frac{k-3}{2}}, \quad (7)$$

(for $k = 3, 5, \dots, \min(N, F)$). Under this mapping, all the continuous global symmetries are preserved. Note however that the baryon B_1 is an elementary field in the dual description.

Consider giving an expectation value of rank one to M ; for definiteness $\langle M_{F, N+F-4} \rangle \neq 0$. The electric theory is higgsed to $SU(N-1)$. The result is a dual description for $SU(N_c)$ with an antisymmetric tensor and an even number of colors N_c . Then add a term proportional to $B_{1,F} \equiv \text{Pf } \hat{A}$ to the superpotential of the electric theory. The effect on these perturbations on the dual is more easily seen on the second dual (5), for which the superpotential is cubic. The effect is to give a mass to q_{N+F-4} and \bar{q}_F and to higgs the $SU(F-3)$ part of the gauge group to $SU(F-4)$ by giving an expectation value $\langle p\bar{q}_F \rangle \neq 0$. The fields B_1 , $M_{F,i}$, $M_{j, N+F-4}$, p , \bar{q}_F are massive. After integrating them out, one recovers precisely the first of Berkooz' duals [15]. His other dual is obtained by dualizing the $Sp(\frac{N-3}{2})$ gauge group.

It is straightforward to check that the duality between the theories (1) and (2) reduces to the duality of [1] for $N = 3$. Since the antisymmetric tensor A is a $\bar{3}$ of $SU(3)$, we have a larger flavor symmetry $SU(F) \times SU(F)$, with F flavors of 3 and $\bar{3}$. In the dual (2), the

$Sp(F-4)$ gauge group has $F-2$ flavors and thus confines [9], yielding operators $\overline{H} \equiv l^2$, $a \equiv x^2$, $q \equiv xl$ and a confining superpotential $\sum_{n=0} a^n \overline{H}^{n+1} q^{F-3-2n}$. The superpotential of the magnetic theory has also the piece $Mq\overline{q} + H\overline{H} + B_1 p\overline{q} + a\overline{a}$. The fields a , \overline{a} , H , \overline{H} are all massive, and after they are integrated out, there remains only $Mq\overline{q} + B_1 p\overline{q} = \hat{M}\hat{q}\hat{\overline{q}}$ for $\hat{q} = (q, p)$, $\hat{\overline{q}} = \overline{q}$, $\hat{M} = (M, B_1)$, which is precisely the $SU(F-3)$ dual of $SU(3)$ with F flavors of [1] with the correct superpotential.

Another special case is for $F = 5$. Then the dual is $SU(2) \times SU(2)$ and the field \overline{a} is just a singlet that is identified with the baryon B_5 of the electric theory.

We now analyze the baryonic flat directions $\langle B_k \rangle \neq 0$. The electric theory becomes $Sp(\frac{N-k}{2})$ with $N-k+2F-4$ fundamentals as mentioned above. When $k = 1$ and $F > 5$, the effect on the magnetic description is to provide a new dual (though possibly a very strongly coupled one) for Sp gauge theories with fundamentals. For $k > 1$, $\langle \tilde{B}_{F-k} \rangle = \langle \overline{q}^{F-k} \overline{a}^{\frac{k-3}{2}} \rangle \neq 0$ in the dual. The $SU(F-3)$ group is higgsed to $Sp(\frac{k-3}{2})$ with $k+1$ fundamentals \overline{q} and p remaining since the fundamentals x are made massive by the $\overline{a}xx$ superpotential. This $Sp(\frac{k-3}{2})$ gauge group confines, yielding a confining superpotential $\overline{q}^k p$ and the fields $\overline{B}_1 = \overline{q}p$ and B_1 are massive. The massless fields that remain and that transform under the $Sp(F-4)$ gauge group are $F-k$ fundamentals x and $N+F-4$ fundamentals l of $Sp(F-4)$. After integrating out the massive fields, the superpotential for the light fields is $Mxl + Hll + \overline{a}xx$ where an expectation value $\langle \overline{q} \rangle$ has been absorbed in a redefinition of M . Rewriting $\hat{M} = \begin{pmatrix} H & M \\ -M & \overline{a} \end{pmatrix}$ and $\hat{q} = (l, x)$, we see that the theories are precisely dual under the $Sp(N_c)$ duality of [1]. When $F = 5$ and $k = 1$, after a similar analysis, one recovers the Sp duality of [1]. At first sight, this magnetic theory for $F > N$ seems to have more operators than the electric theory, namely the \tilde{B}_{F-k} for $k > N$. However, whenever these operators are non-vanishing, there is no supersymmetric vacuum: the $Sp(F-4)$ theory generates dynamically for \hat{q} a superpotential which slopes to infinity for $k > N+2$, or a constraint $\text{Pf } \hat{q} = \Lambda^{2(F-3)}$ for $k = N+2$; but the equations of motion cannot be satisfied since the equation of motion for \hat{M} sets $\hat{q}\hat{\overline{q}}$ to zero. Therefore the magnetic theory has the same chiral ring as the electric theory.

Consider giving an expectation value to \overline{B} . The electric theory is completely higgsed. There remains NF fields Q , $\frac{N(N-1)}{2}$ fields A and $N(F-4)$ fields \overline{Q} as massless singlets. In the magnetic theory $\langle p(xl)^{F-4} \rangle \neq 0$. The $SU(F-3) \times Sp(F-4)$ group is completely higgsed. The fields p , x and $(F-4)^2$ of the fields l are eaten. The superpotential makes B_1 , \overline{q} , \overline{a} as well as some components of l , M and H massive. The massless components of

l , M and H that remain correspond to the singlets \overline{Q} , Q and H respectively, and, as in the electric theory, there is no superpotential for the light fields.

The $SU(N)$ theory with an antisymmetric tensor studied in this letter is a chiral theory and is thus susceptible to break supersymmetry dynamically [4,19,20]. The question of SUSY breaking is analyzed here from the point of view of the magnetic theory. One considers the theory for N odd and $F = 5$. One first integrates out one flavor of Q and \overline{Q} to leave the non-Abelian Coulomb phase. The magnetic theory is completely higgsed and very weakly coupled. It consists of the fields M , H , B_1 , B_3 and \overline{B} and a superpotential proportional to $B_1 M^3 H^{(N-3)/2} + B_3 M H^{(N-1)/2} + \overline{B} B_1 B_3$ which describes the theory everywhere on the moduli space. The last term comes from the term $B_1 p \overline{q}$ of the magnetic superpotential. We included the first two terms to agree with the results described earlier, even though we do not know the actual origin of these terms. From this $F = 4$ theory, it is straightforward to show that the theory with $F = 0$ breaks SUSY dynamically. We add a rank 4 mass term $\sum_{i,j=1}^4 m^{ij} Q_i \overline{Q}_j$ and Yukawa couplings $\lambda A \overline{Q} Q$ over all the \overline{Q} s except \overline{Q}_i , $i = 1, \dots, 4$ (i.e. λ has rank $N - 1$). This lifts all the classical flat directions of the electric theory. Now study the effect of these perturbations on the dual. First $B_1 B_3 = 0$ by the equation of motion for \overline{B} . Then, multiplying the M equation of motion by B_1 , we get that $B_1 = 0$; thus $m = H^{k-1} B_3$; multiplying this equation by B_3 , it says that $B_3 = 0$, implying $m = 0$, a contradiction. Thus dynamical supersymmetry breaking occurs at tree level in the dual description.

Related results about SUSY breaking in the dual description are discussed in [22,11].

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